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Application of bouc-wen model to frequency-dependent nonlinear hysteretic friction damper †

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Abstract

To reduce vibration and noise, various dampers have been applied to many devices such as vehicle suspensions and washing machines. In washing machines, lubricated friction dampers, usually of very simple structure, are installed. Despite the simple structure of these dampers, their dynamic behaviors do not lend themselves to easy mathematical model development, owing to nonlinear characteristics such as hysteresis, viscous elasticity, and other complications. To determine the characteristics of a lubricated friction damper, physical tests with various amplitudes and frequencies were carried out. The complicated lubricated friction damper curves obtained through the physical tests showed hysteretic behavior. In the present study, an analytical model of a hysteretic friction damper was developed, after which optimum parameter values and the maximum friction force were evaluated using an optimization technique. For description of the friction force as changed by the excitation amplitudes and frequencies, a relationship between the sponge velocity and the STV (stick transition velocity) was considered. To describe the hysteretic behavior of the friction damper, a Bouc-Wen model was adopted.

Keywords: Friction damper; Physical test; STV (stick transition velocity); Bouc-Wen model; Washing machine

1. Introduction

In order to decrease vibration and noise, several types of dampers have been developed and applied to many devices. In vehicle suspensions, for example, a semi-active control device, a magnetorheological (MR) damper, has been used to reduce excitation force transferred to the chassis. Bouc-Wen models have been employed for mathematical modeling of the MR damper's nonlinear behavior and for characterization of its performance [1].

In a drum-type washing machine, when a drum is rotated at high speed in dehydration mode, a high-speed tub is activated, presenting a serious danger of tub-cabinet impact. A friction damper normally is applied to prevent any such impacts. The friction damper exerts different friction forces depending on the stick behavior in the contacting surface.

Several models of friction damper have been proposed [2-5]. Liang [2] simply adopted a Coulomb friction model, and Wee [3] analyzed a two-dimensional stick and slip model of the shear force occurring in the rubber bushing. Ok [4] analyzed frequency and amplitude characteristics of the rubber bushing with a Bouc-Wen model. Yoo [5] suggested a MSTV friction model for determination of the turning point from transient friction force to maximum friction force.

In the present study, a Bouc-Wen model was utilized to develop an analytical model of a friction damper used in a washing machine containing a lubricated sponge. Since the friction force is produced between the sponge and a contacting surface when the damper strokes under arbitrary amplitude and frequency, it can result in nonlinear characteristics over the entire friction damper system, and can have quite different features under external excitation conditions. Accordingly, a Bouc-Wen model was employed to describe the nonlinearities of the friction damper, and its results were compared with those of other models.

2. Test of friction damper

2.1 Structure of friction damper

A cross-section of a friction damper used in washing machines to reduce vibration and noise is shown in Fig. 1. The outer tub (1), inner tub (2), sponge (3), rubber bushing (3) and orifice (5) are indicated. The sponge, located between the outer and inner tubs, generates friction force on the contacting surface. During damper motion, air resistance is generated by air flow through the orifice. The rubber bushing helps to con-

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Table 1. Frequency and displacement in harmonic input.

Frequency f [Hz]	Displacement δ_i [mm]
1.0	0.3, 0.5, 1.0, 5.0, 10.0
3.0	0.3, 0.5, 1.0, 3.0, 5.0
6.67	0.3, 0.5, 1.0, 1.5
10.0	0.3, 0.5, 1.0, 1.5

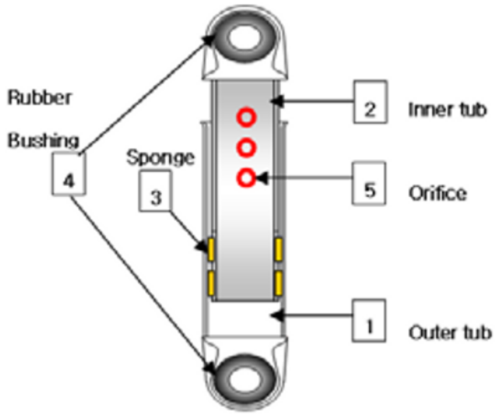


Fig. 1. Structure of friction damper.

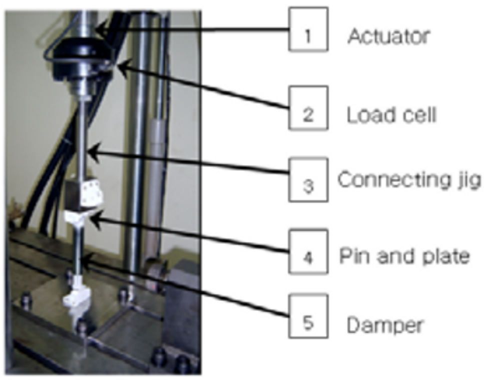


Fig. 2. Damper with jig in testing system.

nect the damper to the washing machine tub and, thereby, to reduce vibration.

2.2 Test procedure

A uni-axial testing system was installed with a damper connected to a jig to determine the characteristics of the friction damper, as shown in Fig. 2. To determine the stiffness and damping characteristics of the friction damper, several harmonic inputs, shown in Table 1, were used. The maximum frequency, 10 Hz, was selected according to the maximum speed of the drum during the dehydration process. The amplitude was selected according to the actual displacement of the damper.

2.3 Test results

Figs. 3 and 4 show the F-D and F-V curves of the excitation tests carried out for the operational frequencies and a fixed displacement of 1 mm, respectively. As Fig. 3 illustrates, as the applied frequency increases, the measured maximum damping force also increases, generally. This feature of the friction damper's frequency-dependent hysteretic behavior is shown more clearly in Fig. 4, where it is apparent that as the applied frequency increases, the measured maximum damping force increases and the hysteretic loop is nonlinearly changed.

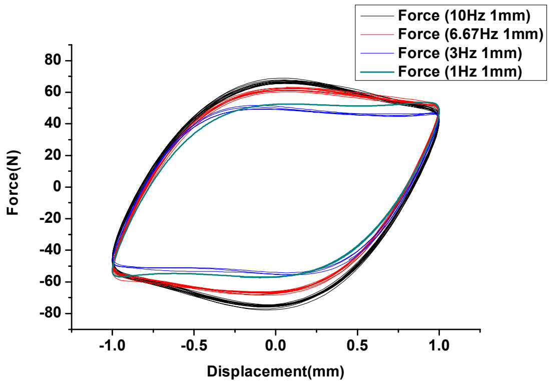


Fig. 3. F- D curve versus various frequencies.

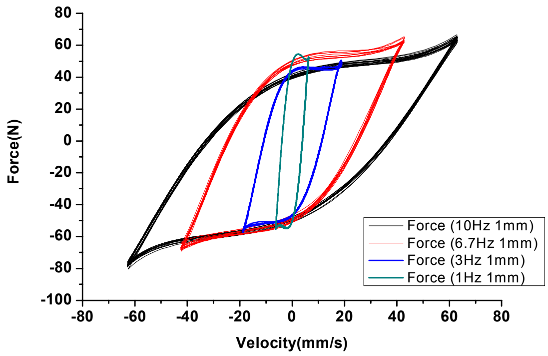


Fig. 4. F-V curve versus various frequency.

3. Modeling of friction damper

3.1 Mathematical model of friction damper

The damper system shown in Fig. 1 was modeled as a combination spring and damper (see Fig. 5) in order to develop a mathematical model. In Fig. 5, Y(t) and X(t) are the external displacement and the displacement of the sponge, respectively. The two masses M and m represent the mass of the jig and the mass of the moving part in the damper, respectively. The force due to deformation of the sponge is assumed in the form of a linear spring and damper, which has a stiffness of k_1 and a damping coefficient of c_1. The air resistance produced by the airflow through the orifice is modelled as a damper with a damping coefficient of c_2. The force F_{fric} represents the friction force between the sponge and the outer tub.

The equations of motion for the system shown in Fig. 5 can be derived as shown in Eq. (1) and Eq. (2), where F(t) is the force applied on the jig to activate the vibration.

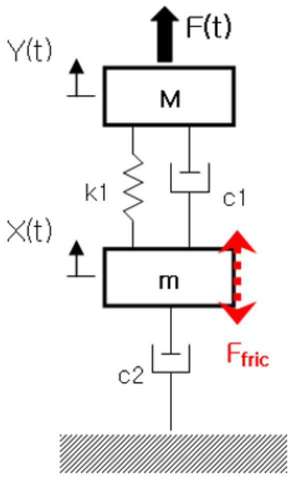


Fig. 5. Mathematical model of friction damper.

Since the coefficients k_1, c_1 and c_2 are assumed as linear coefficients, the frictional damping force F_{fric} should describe all of the hysteretic features of the damper system shown in Fig. 3 and Fig. 4. Therefore, in order to model a hysteretic frictional damping force F_{fric}, the following nonlinearity of the friction damper was considered.

3.2 Nonlinearity of friction damper

When the friction damper is stroked by excitation force, the lubricated sponge shows viscous elastic behavior, as shown in Fig. 6. In the initial state, (Fig. 6(a)), the sponge has no stick and slip motion. Due to the shear force generated by the external excitation force, the sponge could have either stick motion or both stick and slip motion. If the shear force is smaller than the friction force (Fig. 6(b)), the sponge has just stick motion, which means that the magnitude of the friction force depends on that of the shear force. Or, if the shear force is larger than friction force (Fig. 6(c)), the sponge has both stick and slip motion. In this case, a transient friction force or a maximum friction force is exerted on the contacting surface and the deformation velocity of the sponge, $V_{deform.}$, accords with the magnitude of the excitation frequency.

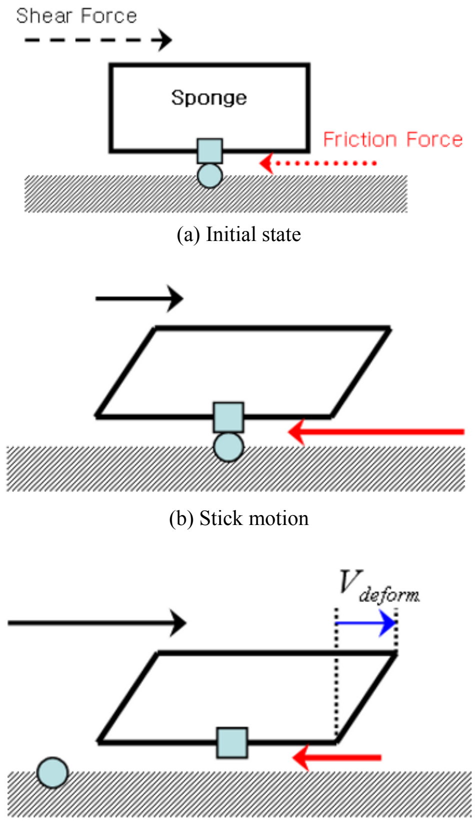
3.3 Models for nonlinear friction damper

3.3.1 STV (stick transition velocity) model

In order to apply a suitable friction force to the mathematical friction damper model shown in Fig. 5, the sponge velocity, which represents the transition point from stick to slip motion, should be identified.Furthermore, due to the various conditions of the external excitation acting on the friction damper, the transition point should be changed by applying excitation frequency and amplitude, as shown in Fig. 7.

$$M\ddot{Y}(t) + c_1\dot{Y}(t) + k_1 Y(t) = F(t) + c_1\dot{X}(t) + k_1 X(t) \tag{1}$$

$$m\ddot{X}(t) + (c_1 + c_2)\dot{X}(t) + k_1 X(t) = c_1\dot{Y}(t) + k_1 Y(t) + F_{fric} \tag{2}$$



(a) Initial state

(b) Stick motion

(c) Stick & Slip motion

Fig. 6. Concept of stick & slip motion.

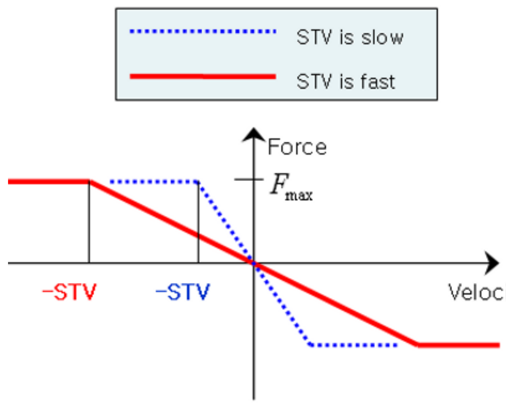


Fig. 7. Various STV friction models.

In the STV (stick transition velocity) model, a relationship between the STV, $\left|\dot{Y}(t) - \dot{X}(t)\right|$, and the absolute sponge velocity, sponge, $\left|\dot{X}\right|$, was used to evaluate the friction force in an arbitrary moment.

$$\left|\dot{Y} - \dot{X}\right| - \left|\dot{X}\right| \begin{cases} > 0 & (3) \\ \le 0 & (4) \end{cases}$$

When the STV is larger than the absolute sponge velocity, as in Eq. (3), stick motion and slip motion occur, and transient friction force is generated. Or, when pure slip motion occurs mainly, as in Eq. (4), the maximum friction force is generated.

3.3.2 MSTV (modified STV) model

For description of the nonlinear behavior of the friction damper as a function of the applied excitation frequency, the MSTV (modified STV) friction model has been suggested [5]. In this model, the relationship between the absolute sponge velocity and the STV was used. If the sponge velocity is larger than the STV, maximum friction force is generated, but if the sponge velocity is smaller than the STV, transient friction force is generated, as shown by calculation of Eq. (5) and Eq. (6):

$$F_{fric.} = \begin{cases} -\mathrm{sgn}\left|\dot{X}(t)\right| F_{\max} & when \ \left|\dot{X}(t)\right| \geq \left|\dot{Y}(t) - \dot{X}(t)\right| \quad (5) \\ -\dfrac{\dot{X}(t)}{\left|\dot{Y}(t) - \dot{X}(t)\right| + \varepsilon} F_{\max} & when \ \left|\dot{X}(t)\right| < \left|\dot{Y}(t) - \dot{X}(t)\right| \quad (6) \end{cases}$$

where $\varepsilon = 0.001$

When the transient friction force is evaluated using Eq. (6), a very small ε value is assigned and used to avoid the denominator becoming zero when $\left|\dot{Y}(t) - \dot{X}(t)\right|$=0. This means minimum value which is not influenced to Eq. (6). Furthermore, the transient friction force would be not only linear but also nonlinear. To overcome this weakness of the MSTV friction model, a Bouc-Wen model was suggested and adopted for the purposes of the present study.

4. Development of bouc-wen friction model

4.1 Mathematical model for Bouc-Wen friction model

The Bouc-Wen model (Fig. 8) has been widely adopted to describe the responses of hysteretic materials and structures to applied force. The general model's equation is Eq. (7). Accordingly, to describe the hysteretic behavior of the friction force, a critical factor Z was applied to the friction damper model as shown in Fig. 9 [6].

A mathematical model of Bouc-Wen friction was formulated as Eq. (8), where α is a constant that controls the magnitude of friction force, and the variable z is evaluated by means of a Bouc-Wen differential equation solution. The Bouc-Wen friction model then was applied to calculate the transient friction force. With this model, the nonlinear transient friction force could be described easily and without need of the small ε value required in the MSTV friction model.

$$F = c_0 \dot{x} + \alpha k_0 x + (1-\alpha) k_0 z \quad (7)$$

$$F_{fric.} = \alpha z \quad (8)$$

where $\dot{z} = A\dot{x} - \beta|\dot{x}|z|z|^{n-1} - \gamma\dot{x}|z|$

4.2 Bouc-Wen model parameter identification

To choose the parameters for use in the Bouc-Wen model shown in Fig. 8, the following two-stage optimization was

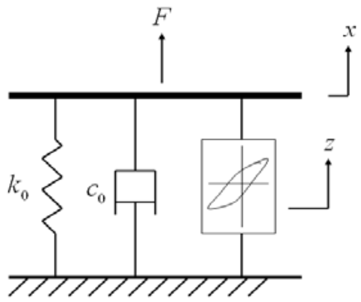


Fig. 8. Bouc-Wen hysteretic model.

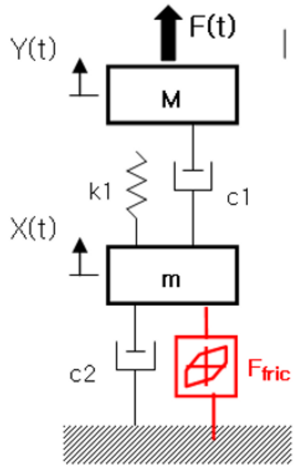


Fig. 9. Mathematical model of friction damper with Bouc-Wen friction model.

carried out. First, the coefficients k1, c1, and c2 were identified assuming the friction forces to be Coulomb friction, which does not contain a hysteresis loop. Then, the detailed Bouc-Wen model parameters for representation of the hysteretic behavior were identified.

4.2.1 Identification for spring and damper

The Coulomb friction model shown in Fig. 10 was assumed for the friction force, which was written in the form of Eq. (9). The coefficients k1, c1, and c2 of the mathematical model in Fig. 8 and the maximum friction force $F_{\max}$ in Eq. (9) were determined using the optimization technique.

$$F_{fric} = -\mathrm{sgn}(\dot{X}) F_{\max} \quad (9)$$

To minimize the difference between the measured data and analytical data, the following Eq. (10) was used as an objective function:

$$\text{Minimize} \quad E = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left[F_{\exp} - F_{cal}\right]^2} \quad (10)$$

where n is the number of data points, and $F_{\exp}$ and F_{cal}

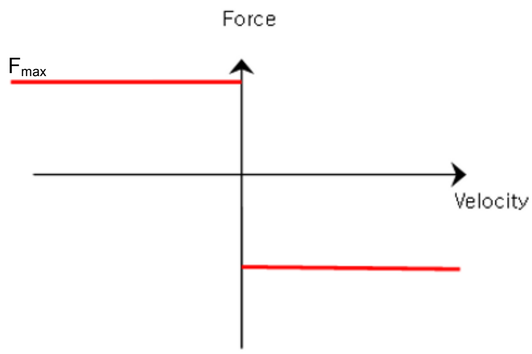


Fig. 10. Coulomb friction model.

represent the experimentally measured force and the calculated force, respectively. To minimize the error function, the VisualDOC program [8] was combined with the ADAMS program [9]. VisualDOC can perform linear, non-linear, constrained and unconstrained as well as integer, discrete and mixed optimization.

Gradient-based, non-gradient based, and response surface approximate optimization algorithms also are available. In the present study, the genetic algorithm (GA) was adopted to solve this identification problem [4]. The obtained stiffness and damping coefficients were optimized for 10 Hz frequency and 1 mm amplitude.

The parameters obtained with the Coulomb model are shown in Table 2. Since the coefficients k_1 , c_1 and c_2 were adopted as linear coefficients, the mathematical model could not represent the hysteretic behavior with them. Thus, the frictional damping force F_{fric} should describe all of the hysteretic features of the damper system. Accordingly then, the Bouc-Wen friction model was considered for modeling of the hysteretic frictional force F_{fric} .

4.2.2 Identification for Bouc-Wen coefficients

For optimization of the Bouc-Wen parameters, the same objective function, Eq. (10), was selected. The range of design variables and identified parameter values optimized for 10 Hz frequency and 1 mm amplitude are listed in Table 3.

The F-V curves are compared with the optimized Bouc-Wen model parameters in Figs. 11 and 12. The results showed good agreement for 10.0 Hz, but a larger discrepancy for 1 Hz.

Table 2. Identified parameters of Coulomb model.

Coeff. & F_{max}	Range	Optimized value
k1 [N/mm]	50~300	114.67
c1 [Ns/mm]	0~10	0.53
c2 [Ns/mm]	0~10	0.49
F_{max} [N]	40~65	44.32

Table 3. Identified parameters of Bouc-Wen model.

Coefficient	Range	Optimized Value
A	10~200	188.34
α	5~50	35.91
β	5~70	38.93
γ	-30~30	29.36
n	1.1~4	3.51

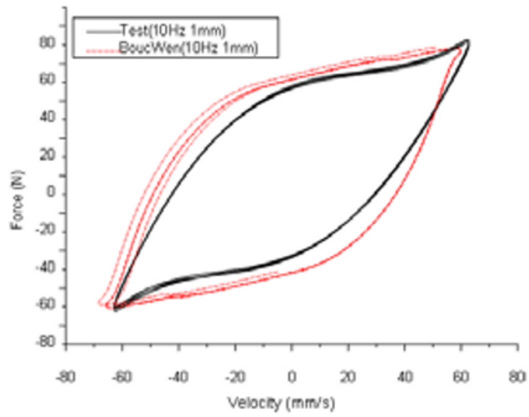


Fig. 11. Comparison of F-V curves (10 Hz, 1 mm).

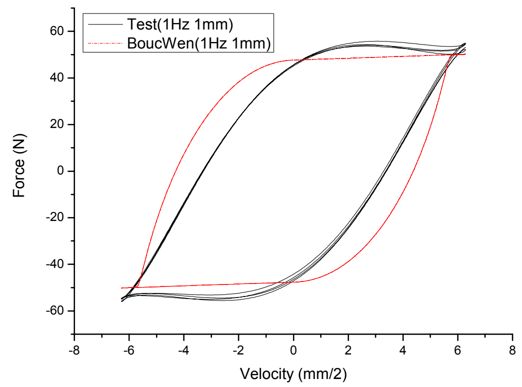


Fig. 12. Comparison of F-V curves (1 Hz, 1 mm).

4.3 Comparison of Bouc-Wen model with MSTV model and test results

The MSTV friction model [5] and real test results were compared with the developed Bouc-Wen friction model in order to determine the accuracy of the latter for 10 Hz frequency and 1 mm amplitude, as shown in Fig. 13. The Bouc-Wen model showed good agreement with real test results, except for two regions. The errors in regions AB and CD were assumed to have occurred because the mathematical model could not catch, in the real system, all of the factors around extreme positions where maximum forces occur.

With the parameters thus optimized, several additional cases were compared. Figs. 14, 15 and 16 show the results for frequencies of 6.67 Hz, 3 Hz and 1 Hz when the amplitude was fixed at 1 mm. The performances in the two cases shown in Figs. 12 and 13 were in roughly good agreement with the real test results and similar to the MSTV model. Since the parameters were optimized based on the experimental results

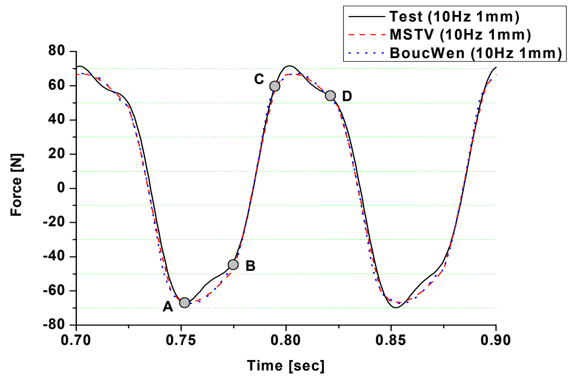


Fig. 13. Bouc-Wen versus MSTV and test (10 Hz, 1 mm).

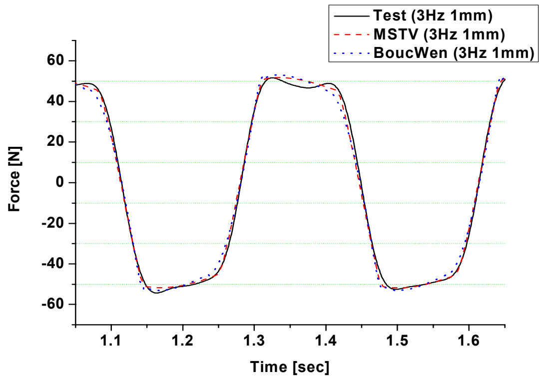


Fig. 15. Bouc-Wen versus MSTV and test (3 Hz, 1 mm).

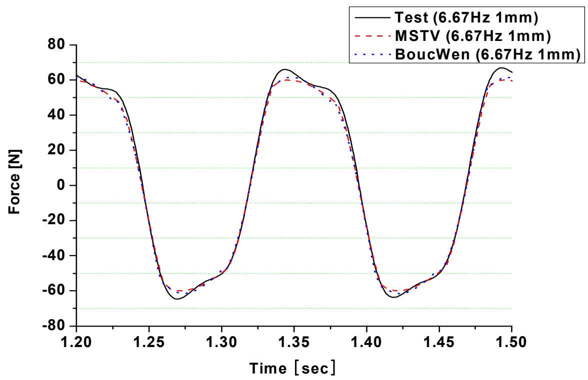


Fig. 14. Bouc-Wen versus MSTV and test (6.67 Hz, 1 mm).

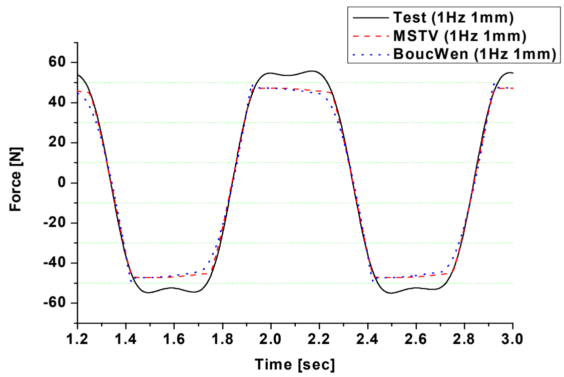


Fig. 16. Bouc-Wen versus MSTV and test (1 Hz, 1 mm).

for 10 Hz, there was almost no discrepancy for the Fig. 13 case (10 Hz). As shown in Fig. 14 however, when the frequency was 6.67 Hz the discrepancy grew slightly larger. With the frequency of 1 Hz (Fig. 16), the discrepancy was the largest, but still less than 10%.

5. Conclusions

In the present study, a Bouc-Wen model was utilized to develop an analytical model of a hysteretic friction damper containing a lubricated sponge. To develop the Bouc-Wen model, first, physical tests with various amplitudes and frequencies were carried out for the friction damper. From the physical test results, the maximum damping force and the hysteretic behavior of the friction damper were measured and evaluated.

To describe these nonlinearities of the friction damper, the stick and slip motion of the sponge was verified, and previous models were reviewed. The stick and slip motion of the sponge could result in rather complicated force, for which the STV (stick transition velocity) and MSTV (modified STV) models were reviewed. Comparing these damping models with the Bouc-Wen model, the latter was considered to overcome the disadvantages of the former two.

To identify the Bouc-Wen model parameters, the spring and damping coefficients were first identified without considering hysteretic behavior. Then, the parameters and design variables of the Bouc-Wen friction model were optimized for the case of 10 Hz operational frequency and 1 mm amplitude. Then, the performance of the Bouc-Wen friction damper model was compared with the test results for several frequencies. The results with the developed Bouc-Wen model showed roughly good agreement with those of the real tests.

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